Plug and Play Distributed Model Predictive Control for Heavy Duty Vehicle Platooning and Interaction with Passenger Vehicles

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***Abstract*— Heavy duty vehicle (HDV) platooning has been widely accepted as a solution to reduce fuel consumption and traffic congestion. However, the control strategy for HDV platoons interacting with other vehicles is not yet well established. This work presents a new framework for handling the requests of passenger vehicles (PV) plugging in or out of an HDV platoon. It consists of three main steps. First, the basic cruising control of the platoon is achieved by a distributed model predictive control (DMPC) scheme. Second, redesign of controllers ensures the stability of closed loop Plug and Play (P&P) operation. Finally, a transition phase to steady-states guarantees the feasibility of the newly synthesized controllers. Additionally for the plug-in case, we propose a novel approach of Formation Coordinator that determines the optimal location at which the redesigned controller has the best initial feasibility. The performance of the proposed control framework is illustrated on a multi-vehicle platooning system.**

I. INTRODUCTION

As the issue of traffic congestion arises globally, vehicle platooning has been widely studied and recognized as a promising technique to increase the road throughput. Earlier research such as [1]–[3] provided the theoretical foundation of vehicle platooning as a control problem. Since then, a considerable body of work has been developed for the platooning of ground vehicles [4]. Today, the concept of vehicle platooning even extends to drones and unmanned air traffic systems [5], [6].

In modern intelligent transportation systems, platooning has the additional advantage of improved fuel efficiency due to reduced air drag, especially when applied to heavy duty vehicles (HDVs) [7]–[11]. In [12], predictive controllers are designed for fuel optimal regulation of HDV platoon systems, with stability guarantees.

Fuel efficiency and stability are two important aspects that one must account for in HDV platooning control. However, HDV convoys usually travel on highways in the presence of other vehicles, mostly passenger vehicles (PV). This brings many potential interactions between HDV platoons and highway traffic [13]. For example, a long HDV platoon may obstruct a PV that wants to change its lane from one side of the platoon to the other, causing the PV to, for instance, miss a highway exit. One method of addressing this challenge is for the PV to "plug-into" the platoon, temporarily becoming part of it, and then "plug-out" from the platoon to the other side, as is depicted in Figure 1.

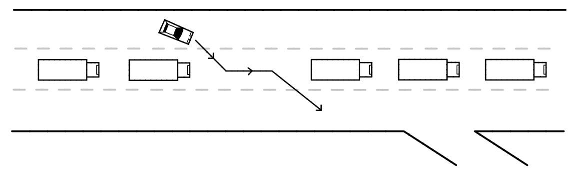

Fig. 1: A typical HDV platooning scenario with a passenger vehicle plugging in and out for lane changing.

Such a plug-and-play (P&P) scenario is challenging for several reasons. First, the HDV platoon must change its formation and thus a feasible controller for transitioning between two formations is needed. Second, in such a safety-critical environment, guarantees of stability and collision avoidance are necessary. Lastly, the vehicles in a long platoon need to be controlled in a distributed manner due to the varying state ability mentioned in [14], which is caused by occasional delay or failure in long-range communication.

Projects such as [15] have explored several controllers for performing various platoon maneuvers. However this framework does not give the guarantee of constraint satisfaction at all times. Model predictive control (MPC) techniques provide us a powerful tool for constraint satisfaction. Several projects are developed based on MPC for P&P control, e.g. [16] is based on distributed MPC (DMPC) and [17] is based on robust MPC. We propose to use the P&P DMPC in [16] for HDV platooning control, mainly because it effectively overcomes the limited communication ability and well handles the interaction with other vehicles.

Statement of Contributions:

In this paper, we consider a platoon of HDVs, with a PV plugging into it, resulting in a new, longer platoon. Building on the P&P DMPC controller described in [16], we develop distributed controllers for basic vehicle movements within a platoon. Motivated by the controller redesign procedure in P&P DMPC, we introduce the novel approach of Formation Coordinator, for the HDV platoon, to effectively choose a suitable new formation that makes the plug-in operation requested by the PV feasible. A transition phase tailored for the HDV platooning system is enabled to guarantee the recursive feasibility of the redesigned DMPC controllers. Crucially, using our proposed control framework, all the necessary platoon maneuvers for handling P&P requests can be fully automated, and guarantees of collision avoidance are achieved at all times.

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The rest of the paper is organized as follows: Section II reviews the P&P DMPC scheme. Section III presents the platoon models. In Section IV P&P control strategies for HDV platooning are derived. Section V illustrates the performance of our proposed framework using simulation examples. Finally, Section VI concludes the paper.

II. Preliminaries

In [16], a P&P DMPC scheme is introduced for a network of interconnected linear systems. It allows for network modification during closed loop operations, while maintaining stability and recursive feasibility at all times.

In the literature of P&P DMPC, the controlled global system is assumed to be linear time-invariant. Without loss of generality, we only consider the discrete-time formulation:

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k, \tag{1}$$

with state vector $\mathbf{x} \in \mathbb{R}^n$ and input vector $\mathbf{u} \in \mathbb{R}^m$. Suppose the global system consists of M dynamically coupled subsystems, and each has the following dynamics:

$$x_{k+1}^{[i]} = \sum_{j\in\mathcal{N}_i} A_{ij}x_k^{[j]} + B_i u_k^{[i]} \ , \tag{2}$$

where $x^{[i]} \in \mathbb{R}^{n_i}$ and $u^{[i]} \in \mathbb{R}^{m_i}$ denote the state and the input of subsystem i, $i \in \mathcal{M} := \{1,\ldots,M\}$, then $\mathbf{x} = [x^{[1]},\cdots,x^{[M]}]$ and $\mathbf{u} = [u^{[1]},\ldots,u^{[M]}]$. $\mathcal{N}_i$ denotes the *neighborhood* of subsystem i, including i itself, defined as $\mathcal{N}_i := \{j \in \mathcal{M} \ | A_{ij} \neq 0\}$. We denote by $x^{\mathcal{N}_i}$ the concatenated vector of states of all subsystems $j \in \mathcal{N}_i$.

A. Distributed MPC

The P&P scheme proposed in [16] is based on the DMPC approach presented in [18], which is formulated as follows:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{M}\sum_{k=0}^{N-1} \|x_k^{[i]}\|_{Q_i}^2 + \|u_k^{[i]}\|_{R_i}^2 + \|x_N^{[i]}\|_{P_i}^2 && \text{(3a)}\\
\text{s.t.} \quad & x_0^{[i]} = x^{[i]}(t), && \text{(3b)}\\
& x_{k+1}^{[i]} = \sum_{j\in\mathcal{N}_i} A_{ij}x_k^{[j]} + B_i u_k^{[i]}, && \text{(3c)}\\
& x_k^{[i]} \in \mathbb{X}^{[i]}, u_k^{[i]} \in \mathbb{U}^{[i]}, && \text{(3d)}\\
& \|x_N^{[i]}\|_{P_i}^2 \leq \alpha^{[i]}(t), && \text{(3e)}\\
& \forall k = 0,\ldots,N, \forall i = 1,\ldots,M.
\end{aligned}$$

where the sets $\mathbb{X}^{[i]} := \{x^{[i]} | G_x^{[i]} x^{[i]} \leq f_x^{[i]}\}$ and $\mathbb{U}^{[i]} := \{u^{[i]} | G_u^{[i]} u^{[i]} \leq f_u^{[i]}\}$ are polytopic constraints on local states and inputs. $\|\cdot\|_{P_i}^2$ are the local terminal cost functions. Optimization problem (3) returns for each subsystem i the optimal input sequence $[u_0^{[i]*},\ldots,u_{N-1}^{[i]*}]$ and applies the control law $\kappa^{[i]}(\mathbf{x},\alpha^{[1]},\ldots,\alpha^{[M]}) = u_0^{[i]*}$ in a receding horizon fashion. The local terminal constrains (3e) are time-varying level sets of the local terminal cost functions with the size $\alpha^{[i]}(t)$. As is discussed in [18], to ensure stability, the size $\alpha^{[i]}(t)$ is updated at each time step t, according to the following set-updating dynamics:

$$\alpha^{[i]}(t+1) = \alpha^{[i]}(t) + x_N^{\mathcal{N}_i\,T}\Gamma_{\mathcal{N}_i} \cdot x_N^{\mathcal{N}_i}. \tag{4}$$

Note that for given $Q_i \succ 0$ and $R_i \succ 0$ the matrices $\Gamma_{\mathcal{N}_i}$, P_i, and the sum of the initial size $\alpha := \sum_{1\leq i\leq M}\alpha^{[i]}(0)$ satisfy the following conditions for all $i \in \mathcal{M}$:

$$\begin{aligned}
&(A_i + B_iK_i)^T P_i (A_i + B_iK_i) - P_i^{\text{lift}} \\
&\leqslant -Q_i^{\text{lift}} - K_i^T R_i K_i + \Gamma_{\mathcal{N}_i}^{\text{lift}}, && \text{(5a)}\\
&\sum_{i=1}^{M}\Gamma_{\mathcal{N}_i} \preceq 0, \ P_i \succ 0, \ \alpha > 0, && \text{(5b)}\\
&G_{x,j}^{[i]} P_i^{-1} G_{x,j}^{[i]\,T} \leq \frac{1}{\alpha} f_{x,j}^{[i]\,2}, \ j = 1,\ldots,p_x^{[i]}, && \text{(5c)}\\
&G_u^{[i]} K_i P^{-1} K_i^T G_u^{[i]\,T}\alpha \leq f_{u,k}^{[i]\,2}, \ k = 1,\ldots,p_u^{[i]}, && \text{(5d)}
\end{aligned}$$

where $A_i = [A_{i1},\ldots,A_{iM}]$, $P = diag(P_1,\ldots,P_M)$ and K_i are helping matrices. The operator $(\cdot)^{\text{lift}}$ denotes the lifting of any local matrix to the global system space. The parameters $p_x^{[i]}$ and $p_u^{[i]}$ denote the number of rows of the vectors $f_x^{[i]}$ and $f_u^{[i]}$.

Based on (5), the optimization problem to be solved for the design of DMPC controllers is formulated as follows:

$$\begin{aligned}
\min_{P_i,K_i,\Gamma_{\mathcal{N}_i},\alpha} \quad & -\alpha \\
\text{s.t.} \quad & \text{(5a), (5b), (5c), (5d).}
\end{aligned} \tag{6}$$

One way to get the terminal set $\alpha^{[i]*}$ for each subsystem is simply by equally dividing α^*, i.e. $\alpha^{[i]*} = \frac{1}{M}\alpha^*$.

Remark II.1. As is shown in [16] and [18], constraints (5a), (5b), (5c), (5d) in (6) can be written as a set of LMIs. Therefore (6) can be reformulated into a convex optimization problem and solved efficiently in a distributed manner.

B. Plug and Play Distributed MPC

The Plug and Play (P&P) method proposed in [16] consists of the following four steps: 1. Send P&P request to neighboring subsystems; 2. Redesign the DMPC controller; 3. Compute and track the steady-state; 4. If P&P is permitted, i.e. Step 2 is feasible and Step 3 is finished, plug-in/-out the new subsystems and apply the modified control law. More precisely, Steps 2 and 3 are introduced as follows.

1) Controller redesign: As a P&P operation changes the dynamics of the global system, we denote by $\tilde{(\cdot)}$ all quantities associated with the modified system. To adapt to those changes, the stability-critical parameters P_i, K_i, $\Gamma_{\mathcal{N}_i}$ and $\alpha^{[i]}$ are re-computed by solving (6), taken as input $\tilde{A}_i$ and $\tilde{B}_i$, matrices of the modified system.

2) Compute and track the steady-state: To ensure that the redesigned controller is initially feasible, we first compute the steady-states x_{ss} for all subsystems, starting from which there exists a feasible trajectory given by the new control law $\tilde{\kappa}^{[i]}(\cdot)$. Then a tracking controller regulates the subsystems to their steady-states. Only when the steady-states are reached by all subsystems, the P&P request can be permitted.

Theorem II.1. [16] The modified closed loop system, starting from the steady-state x_{ss}, under the control law $\tilde{\kappa}^{[i]}(\cdot)$ designed by (6), additionally with the set updating dynamics (4), is asymptotically stable and $\tilde{\kappa}^{[i]}(\cdot)$ is recursively feasible.

III. Platoon Representation and Modeling

Define a platoon with index i, $\mathcal{P}_i$, as an ordered set of vehicles, and suppose there are M_1 HDVs, denoted $\mathcal{Q}_1^{\text{HDV}}, \dots, \mathcal{Q}_{M_1}^{\text{HDV}}$, in the platoon $\mathcal{P}_1$, and M_2 PVs, denoted $\mathcal{Q}_1^{\text{PV}}, \dots, \mathcal{Q}_{M_2}^{\text{PV}}$, in the platoon $\mathcal{P}_2$. Then, as an example, if $M_1 = 4$ and $M_2 = 1$, we have $\mathcal{P}_1 = \{\mathcal{Q}_1^{\text{HDV}}, \dots, \mathcal{Q}_4^{\text{HDV}}\}$, $\mathcal{P}_2 = \{\mathcal{Q}_1^{\text{PV}}\}$, which represent the *formation* of each platoon. More precisely, the physical interpretation of $\mathcal{P}_1$ is a platoon of four HDVs, with $\mathcal{Q}_1^{\text{HDV}}$ being the leader of the platooning, traveling at the front of the platoon, and with each HDV $\mathcal{Q}_i^{\text{HDV}}$ traveling immediately behind $\mathcal{Q}_{i-1}^{\text{HDV}}$ for $i > 1$.

Given a platoon $\mathcal{P}$ and a time index k, we denote the joint state of all vehicles in the platoon as $x_k^{\mathcal{P}}$, and the state of each vehicle in $\mathcal{P}$ as $x_k^{\mathcal{P}[i]}$, where the index i specifies the position of a vehicle in the platoon $\mathcal{P}$.

For convenience, we define the plug-in operation $\mathbb{M}(\cdot,\cdot,\cdot)$, which takes as input two platoons and an index, and outputs the new platoon with modified formation, resulting from the second platoon plugging into the first platoon after the vehicle at the indicated index. For example, using the definition of $\mathcal{P}_1$ and $\mathcal{P}_2$ above, we have:

$$\tilde{\mathcal{P}} = \mathbb{M}(\mathcal{P}_1, \mathcal{P}_2, 2) = \{\mathcal{Q}_1^{\text{HDV}}, \mathcal{Q}_2^{\text{HDV}}, \mathcal{Q}_1^{\text{PV}}, \mathcal{Q}_3^{\text{HDV}}, \mathcal{Q}_4^{\text{HDV}}\}.$$

The plug-out operation can be defined in a similar way.

Remark III.1 (Sample Platoon Formation)**.** We take the following example throughout the paper to explain our methods clearly. For $\mathcal{P}_1$, $\mathcal{P}_2$ and $\tilde{\mathcal{P}}$ we refer to the platoon formation: $\mathcal{P}_1 = \{\mathcal{Q}_1^{\text{HDV}}, \mathcal{Q}_2^{\text{HDV}}, \mathcal{Q}_3^{\text{HDV}}, \mathcal{Q}_4^{\text{HDV}}\}$, $\mathcal{P}_2 = \{\mathcal{Q}_1^{\text{PV}}\}$ and $\tilde{\mathcal{P}} = \{\mathcal{Q}_1^{\text{HDV}}, \mathcal{Q}_2^{\text{HDV}}, \mathcal{Q}_1^{\text{PV}}, \mathcal{Q}_3^{\text{HDV}}, \mathcal{Q}_4^{\text{HDV}}\}$ unless noted otherwise. One may notice that the approaches presented in this paper are well held for more general cases.

Assumption III.1 (Communication)**.** Any two vehicles, i.e. $\mathcal{Q}_i^{\text{HDV}}$ and $\mathcal{Q}_j^{\text{HDV}}$, can communicate with each other only if $i \in \mathcal{N}_j$ or $j \in \mathcal{N}_i$.

We now specialize the notion of *neighborhood* presented in Section II to the context of HDV platooning. Because of the above communication restrictions, we assume that coupling dynamics for any vehicle only exist between itself and the vehicle in front of it, if the vehicle is not the leader. Formally, we have that $\mathcal{N}_1 = \{1\}$ and $\mathcal{N}_i = \{i, i-1\}$ for $i > 1$. As a result, the dynamics (2) can be written as follows:

$$\begin{aligned} x_{k+1}^{\mathcal{P}[1]} &= A_{11}^{\mathcal{P}} x_k^{\mathcal{P}[1]} + B_1^{\mathcal{P}} u_k^{\mathcal{P}[1]}, \\ x_{k+1}^{\mathcal{P}[i]} &= A_{ii}^{\mathcal{P}} x_k^{\mathcal{P}[i]} + A_{i(i-1)}^{\mathcal{P}} x_k^{\mathcal{P}[i-1]} + B_i^{\mathcal{P}} u_k^{\mathcal{P}[i]},\ i > 1. \end{aligned} \tag{7}$$

A. Platoon Modeling

Now we introduce a scheme for constructing the global platoon system, which captures the interconnection between different vehicle subsystems within a platoon.

Expanding (7), the model of a general platoon $\mathcal{P}$ consisting of M vehicles $\mathcal{P} = \{\mathcal{Q}_1, \dots, \mathcal{Q}_M\}$ is given as follows:

$$\begin{bmatrix} x_{k+1}^{[1]} \\ x_{k+1}^{[2]} \\ \vdots \\ x_{k+1}^{[M]} \end{bmatrix} = \underbrace{\begin{bmatrix} A_{11} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & 0 & A_{M(M-1)} & A_{MM} \end{bmatrix}}_{A^{\mathcal{P}}} \begin{bmatrix} x_k^{[1]} \\ x_k^{[2]} \\ \vdots \\ x_k^{[M]} \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & B_M \end{bmatrix}}_{B^{\mathcal{P}}} \begin{bmatrix} u_k^{[1]} \\ u_k^{[2]} \\ \vdots \\ u_k^{[M]} \end{bmatrix}. \tag{8}$$

where A_{ii} and B_i are the matrices drawn from each vehicle subsystem; $A_{i(i-1)}$ matrices describe the coupling dynamics.

B. Heavy Duty Vehicle Modeling

In HDV platooning scenarios, for each vehicle, the control objective is to maintain the reference velocity and reference distance to the preceding vehicle. Therefore, it is reasonable to describe the behavior of an HDV using a linearized model around a set of equilibria, including the desired cruising velocity v_0, relative distance d_0, and engine torque T_0.

Our work follows the current practice of HDV platooning that each vehicle is equipped with a driver for controlling the lateral movement. Only the regulation of longitudinal velocity is automated. We further assume that each HDV always keeps to its lane.

Considering the factors above, we propose to use the discrete-time version of the HDV model in [19]:

1) The leader HDV, $\mathcal{Q}_1^{HDV}$:

$$v_{k+1}^{[1]} = \underbrace{(1 + \tau\Theta_l)}_{A_{11}^{HDV}} v_k^{[1]} + \underbrace{\tau c_e}_{B_1^{HDV}} T_k^{[1]}, \tag{9}$$

2) Follower HDVs, $\mathcal{Q}_i^{HDV}, i > 1$:

$$\begin{aligned} \begin{bmatrix} d_{k+1}^{[i]} \\ v_{k+1}^{[i]} \end{bmatrix} &= \underbrace{\begin{bmatrix} 1 & -\tau \\ \tau\delta & 1 + \tau\Theta \end{bmatrix}}_{A_{ii}^{HDV}} \begin{bmatrix} d_k^{[i]} \\ v_k^{[i]} \end{bmatrix} \\ &+ \underbrace{\begin{bmatrix} 0 & \tau \\ 0 & 0 \end{bmatrix}}_{A_{i(i-1)}^{HDV}} \begin{bmatrix} d_k^{[i-1]} \\ v_k^{[i-1]} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \tau c_e \end{bmatrix}}_{B_i^{HDV}} T_k^{[i]}, \end{aligned} \tag{10}$$

where $v^{[i]}$ is the longitudinal velocity, $T^{[i]}$ is the net engine torque, the only input to each vehicle. For a follower, $d^{[i]}$ is the distance to the vehicle in front.

C. Passenger Vehicle Modeling

For a passenger vehicle, both the lateral and longitudinal movements are automated.

Consider the discrete-time, linearized version of the classical kinematic bicycle model in [20]:

$$\begin{bmatrix} y_{k+1}^{[i]} \\ \varphi_{k+1}^{[i]} \\ d_{k+1}^{[i]} \\ v_{k+1}^{[i]} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \tau v_0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\tau \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A_{ii}^{PV}} \begin{bmatrix} y_k^{[i]} \\ \varphi_k^{[i]} \\ d_k^{[i]} \\ v_k^{[i]} \end{bmatrix}$$
$$+ \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A_{i(i-1)}^{PV}} \begin{bmatrix} y_k^{[i-1]} \\ \varphi_k^{[i-1]} \\ d_k^{[i-1]} \\ v_k^{[i-1]} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \tau \frac{v_0}{L_w} \\ 0 & 0 \\ \tau & 0 \end{bmatrix}}_{B_i^{PV}} \begin{bmatrix} a_k^{[i]} \\ \delta_k^{[i]} \end{bmatrix}, \tag{11}$$

where $d^{[i]}$ is the relative distance to the preceding vehicle in the longitudinal direction, $y^{[i]}$ is the lateral position, $\varphi^{[i]}$ is the yaw angle, and $v^{[i]}$ is the velocity with respect to the rear axes. The inputs include the acceleration $a^{[i]}$ and the steering angle $\delta^{[i]}$. The equilibria $d_0 = d_{ref}$ and $v_0 = v_{ref}$ are chosen the same as the HDV model while all the remaining equilibria states and inputs are 0. For simplicity, we do not consider the effect of air drag in the PV model. However, it only requires minor modifications to the model if one wants to include air fluid dynamics here.

Parameters of the HDV and PV model can be found in Table I. Note that the value of the reference relative distance d_0 differs from the one used in [21].

TABLE I: Parameter Values of the HDV [21] and PV Model [20]

Parameter	Symbol	Value
reference velocity	v_0	19.44 m/s
reference distance	d_0	15 m
reference engine torque	T_0	332.79 Nm
sampling interval	τ	0.1 s
PV wheel base	L_w	2.7 m
HDV coefficients	Θ_l, Θ	$-3.6 \times 10^{-3}, -2.1 \times 10^{-3}$
	δ, c_e	$-2.6 \times 10^{-4}, 1.5 \times 10^{-5}$

IV. Plug and Play Vehicle Platooning and Interaction

In this section we tailor the general P&P DMPC algorithm introduced in Section II to HDV platooning scenarios. The P&P operation now specializes to the action that one or multiple PVs plug-in or plug-out from the HDV platoon, as shown by Figure 1.

When a PV is seeking for any P&P operation, it shall send the P&P request to the platoon, indicating its intention. For the plug-in case, the position for the PV to stay in the platoon is determined by the so called Formation Coordinator. Consequently, local controllers for the modified platoon are redesigned according to the new platoon formation. To ensure the feasibility of the new controller, a steady-state is computed for each vehicle subsystem to track as a reference. The new control law is applied only when all vehicles have reached their steady-states. The above procedure of handling P&P operations for an HDV platoon is summarized in Algorithm 1.

Algorithm 1 P&P MPC for vehicle platooning (plug-in case)

- **Distributed MPC, Before Plug-in:** Both $\mathcal{P}_1$ and $\mathcal{P}_2$ are regulated under their own control law $\kappa^{\mathcal{P}_1}(\cdot)$ and $\kappa^{\mathcal{P}_2}(\cdot)$ given in (3)
- **P&P Request:** Vehicle $\mathcal{Q}_1^{\text{PV}}$ sends plug-in request to the platoon $\mathcal{P}_1$
- **Redesign Phase:**
- $\boldsymbol{\alpha} = \emptyset$ *(Begin Formation Coordinator)*
- **for all** $l \in \{1,2,3\}$ **do**
  - Construct model for $\tilde{\mathcal{P}}_l = \mathbb{M}(\mathcal{P}_1, \mathcal{P}_2, l)$ following Section III-A
  - Compute α_l^* and $\tilde{\kappa}_l(\cdot)$ by (6)
  - $\boldsymbol{\alpha} = \boldsymbol{\alpha} \bigcup \{\alpha_l^*\}$
- **end for**
- $\alpha_{max} = \max\limits_{l \in \{1,2,3\}} \boldsymbol{\alpha}$
- Choose the platoon formation $\tilde{\mathcal{P}}_l$ and control law $\tilde{\kappa}_l(\cdot)$ corresponding to α_{max}
- **Computation of the Steady-state:** Obtain $x_{ss}^{\mathcal{P}_1}$ and $x_{ss}^{\mathcal{P}_2}$ by solving (15)
- **Transition Phase:**
- **if** (15) is feasible **then**
  - **repeat**
    - Run tracking MPC (16), (17) for $\mathcal{P}_1$ and $\mathcal{P}_2$
  - **until** $x^{\mathcal{P}_1} = x_{ss}^{\mathcal{P}_1}$, $x^{\mathcal{P}_2} = x_{ss}^{\mathcal{P}_2}$ and $\bar{d} = \bar{d}_{ss}$
  - Plug-in request permitted
- **else**
  - Plug-in request rejected
- **end if**
- **Distributed MPC:** Switch to the new control law $\tilde{\kappa}_l(\cdot)$, carrying out the plug-in operation.

Remark IV.1. For the plug-out case the procedure is the same as Algorithm 1, with the only exception that the Formation Coordinator is no longer needed.

A. Redesign Phase

1) Formation Coordinator: As a product of controller redesign, the value of α^* given by (6) is an effective certification for choosing the plug-in location. Our simulations show that the problem (15) usually has a better feasibility with a larger value of α^*. Motivated by this observation we introduce a novel decision-making process, the Formation Coordinator, as an additional step in the redesign phase. It carries out the controller redesign process (6) with each formation of the new platoon $\tilde{\mathcal{P}}_l = \mathbb{M}(\mathcal{P}_1, \mathcal{P}_2, l)$, for $l = 1,2,3$, and gathers all the resulting α_l^* in a set $\boldsymbol{\alpha}$. We propose to choose the plug-in location l with the largest α_l^*:

$$\alpha_{max} = \max_{l \in \{1,2,3\}} \alpha_l^*.$$

Remark IV.2. Although the Formation Coordinator calls for (6) to redesign controllers in a distributed manner, there has to be a centralized "node" which decides the platoon formation and publishes it to all vehicles. Similar to the idea in [11], since the Formation Coordinator is not related to any specific vehicle, it can be implemented on any vehicle in the platoon or even in a road-side unit.

In fact, α_{max} is not the only certification for determining the plug-in location. For example, since a PV usually wants to plug into the platoon as quickly as possible for a fast lane change, the platoon formation which yields the shortest transition time might also be desirable.

As an illustrating example, if $\alpha_{max} = \alpha_2^*$, the Formation Coordinator then determines that the plug-in location is 2 in between $\mathcal{Q}_2^{\text{HDV}}$ and $\mathcal{Q}_3^{\text{HDV}}$, as is depicted in Figure 2. The new control law $\tilde{\kappa}(\cdot)$ is chosen as $\tilde{\kappa}_2(\cdot)$ accordingly.

Note that during the redesign phase all the vehicles in $\mathcal{P}_1$ and $\mathcal{P}_2$ continue using the control law $\kappa^{\mathcal{P}_1}(\cdot)$ and $\kappa^{\mathcal{P}_2}(\cdot)$ designed for the their current formation.

B. Computation of the Steady-state

We point out that the control law $\tilde{\kappa}(\cdot)$ redesigned for the modified platoon system $\tilde{\mathcal{P}}$ might not be feasible if initialized with the current states $x^{\mathcal{P}_1}(t)$ and $x^{\mathcal{P}_2}(t)$. This usually occurs in the plug-in case. Nevertheless, the issue can be fixed by computing feasible (potentially non-zero) steady-states for both $\mathcal{P}_1$ and $\mathcal{P}_2$, starting from which the new control law $\tilde{\kappa}(\cdot)$ is guaranteed to be feasible.

At the time when $\mathcal{Q}_1^{\text{PV}}$ sends the plug-in request, its position can be far away from the platoon $\mathcal{P}_1$. Intuitively, the tracking controller used in the transition phase should be able to drive $\mathcal{Q}_1^{\text{PV}}$ to catch up with the platoon. However the platoon model (8) is not able to capture the relative distance between $\mathcal{Q}_1^{\text{PV}}$ and any vehicle in $\mathcal{P}_1$. Therefore we introduce an *augmented state* $\bar{d}$ for measuring the distance between $\mathcal{Q}_1^{\text{PV}}$ and $\mathcal{Q}_2^{\text{HDV}}$ during the transition phase. It satisfies the following dynamics:

$$\bar{d}_{k+1} = \bar{d}_k + v_k^{\mathcal{P}_1[2]}\tau - v_k^{\mathcal{P}_2[1]}\tau \tag{12}$$

At the moment when the transition phase is finished and the P&P operation is about to begin, the states of $\mathcal{Q}_1^{\text{PV}}$, $\mathcal{Q}_2^{\text{HDV}}$ and $\mathcal{Q}_3^{\text{HDV}}$ need to be redefined. For a plug-in operation to be feasible, extra relative distance needs to be provided by $\mathcal{Q}_2^{\text{HDV}}$ and $\mathcal{Q}_3^{\text{HDV}}$ so that there is enough room for $\mathcal{Q}_1^{\text{PV}}$ to stay. Those concerns are addressed by the following equations, and captured by Figure 2,

- Relative distance between $\mathcal{Q}_2^{\text{HDV}}$ and $\mathcal{Q}_1^{\text{PV}}$:

$$d^{\tilde{P}[3]} = \bar{d}_{ss} - d_0, \tag{13}$$

- Relative distance between $\mathcal{Q}_1^{\text{PV}}$ and $\mathcal{Q}_3^{\text{HDV}}$:

$$d^{\tilde{P}[4]} = d_{ss}^{\mathcal{P}_1[3]} - L^{\mathcal{P}_2[1]} - \bar{d}_{ss}, \tag{14}$$

where $L^{\mathcal{P}_2[1]}$ is the approximated length of $\mathcal{Q}_1^{\text{PV}}$, being valid when the yaw angle of the vehicle is small. Note that the superscript $\tilde{\mathcal{P}}[3]$ indicates that after $\mathcal{Q}_1^{\text{PV}}$ plugs into $\mathcal{P}_1$, it will be the third vehicle in the new platoon $\tilde{\mathcal{P}}$.

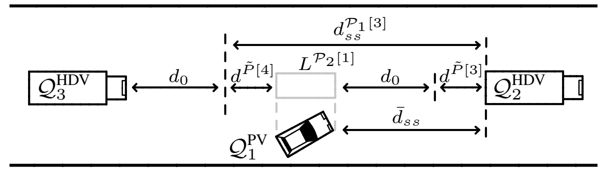

Fig. 2: Redefinition of states at the end of transition phase.

Now we introduce the optimization problem to be solved for computing the steady-state, which is given by:

$$\min_{\substack{\bar{x}_{ss},\bar{x},\bar{u}\\ x^{\bar{\mathcal{P}}},u^{\bar{\mathcal{P}}}}} \sum_{k=0}^{N-1} \|\bar{x}_k\|_Q^2 + \|\bar{u}_k\|_R^2 + \|\bar{x}_{ss}\|_{Q_{ss}}^2 \tag{15a}$$

$$\begin{aligned}
\text{s.t.}\quad & x_{ss}^{\bar{\mathcal{P}}} = A^{\bar{\mathcal{P}}} x_{ss}^{\bar{\mathcal{P}}} + B^{\bar{\mathcal{P}}} u_{ss}^{\bar{\mathcal{P}}}, && (15b)\\
& x_{k+1}^{\bar{\mathcal{P}}} = A^{\bar{\mathcal{P}}} x_k^{\bar{\mathcal{P}}} + B^{\bar{\mathcal{P}}} u_k^{\bar{\mathcal{P}}}, && (15c)\\
& \bar{d}_{k+1} = \bar{d}_k + v_k^{\mathcal{P}_1[2]}\tau - v_k^{\mathcal{P}_2[1]}\tau, && (15d)\\
& \bar{x}_0 = \bar{x}_{init},\ \bar{x}_N = \bar{x}_{ss}, && (15e)\\
& x_k^{\bar{\mathcal{P}}} \in \mathbb{X}^{\bar{\mathcal{P}}},\ u_k^{\bar{\mathcal{P}}} \in \mathbb{U}^{\bar{\mathcal{P}}}, && (15f)\\
& d_N^{\tilde{P}[3]} = \bar{d}_{ss} - d_0\ , && (15g)\\
& d_N^{\tilde{P}[4]} = d_{ss}^{\mathcal{P}_1[3]} - L^{\mathcal{P}_2[1]} - \bar{d}_{ss}, && (15h)\\
& x_{N+k+1}^{\tilde{\mathcal{P}}} = A^{\tilde{\mathcal{P}}} x_{N+k}^{\tilde{\mathcal{P}}} + B^{\tilde{\mathcal{P}}} u_{N+k}^{\tilde{\mathcal{P}}}, && (15i)\\
& x_{N+k}^{\tilde{\mathcal{P}}} \in \mathbb{X}^{\tilde{\mathcal{P}}}, u_{N+k}^{\tilde{\mathcal{P}}} \in \mathbb{U}^{\tilde{\mathcal{P}}}, && (15j)\\
& \|x_{N+N}^{\tilde{\mathcal{P}}[i]}\|_{\tilde{P}_i}^2 \le \alpha^{\tilde{\mathcal{P}}[i]}, && (15k)\\
& k = 1, \dots, N-1,\ \forall \bar{\mathcal{P}} \in \{\mathcal{P}_1, \mathcal{P}_2\}.
\end{aligned}$$

where $\bar{x} := (x^{\mathcal{P}_1}, x^{\mathcal{P}_2}, \bar{d})^T$ and $\bar{u} := (u^{\mathcal{P}_1}, u^{\mathcal{P}_2})^T$.

As is captured by (15b)-(15f), the steady-state $\bar{x}_{ss}$ is chosen such that it can be reached by both platoons $\mathcal{P}_1$ and $\mathcal{P}_2$ from their initial states under current dynamics. (15g)-(15h) redefine states in the new platoon $\tilde{\mathcal{P}}$ before the plug-in operation is carried out. (15i)-(15k) ensure that starting from the steady-state there exists a feasible trajectory when switching to the newly synthesized MPC control law $\tilde{\kappa}(\cdot)$.

If (15) is infeasible, the P&P request shall be rejected. $\mathcal{Q}_1^{\text{PV}}$ has to resend the request or abort the intended maneuver.

C. Transition phase

If (15) returns a feasible solution, both $\mathcal{P}_1$ and $\mathcal{P}_2$ are regulated to the computed steady-states $x_{ss}^{\mathcal{P}_1}$ and $x_{ss}^{\mathcal{P}_2}$ using two tracking MPC controllers, which work in a cooperative manner. The tracking MPC to be solved at time t_t for platoon $\mathcal{P}_1$ is formulated as follows:

$$\min_{x^{\mathcal{P}_1[i]},u^{\mathcal{P}_1[i]}} \sum_{i=1}^{4}\sum_{k=0}^{N-1} \|x_k^{\mathcal{P}_1[i]} - x_{ss}^{\mathcal{P}_1[i]}\|_{Q_i}^2 + \|u_k^{\mathcal{P}_1[i]}\|_{R_i}^2 \tag{16a}$$

$$\begin{aligned}
\text{s.t.}\quad & x_{k+1}^{\mathcal{P}_1[i]} = A_{ii}^{\mathcal{P}_1} x_k^{\mathcal{P}_1[i]} + A_{i(i-1)}^{\mathcal{P}_1} x_k^{\mathcal{P}_1[i-1]}\\
& \quad + B_i^{\mathcal{P}_1} u_k^{\mathcal{P}_1[i]}, && (16b)\\
& x_k^{\mathcal{P}_1[i]} \in \mathbb{X}^{\mathcal{P}_1[i]},\ u_k^{\mathcal{P}_1[i]} \in \mathbb{U}^{\mathcal{P}_1[i]}, && (16c)\\
& x_0^{\mathcal{P}_1[i]} = x^{\mathcal{P}_1[i]}(t_t),\ x_N^{\mathcal{P}_1[i]} = x_{ss}^{\mathcal{P}_1[i]}, && (16d)\\
& \forall k = 0, \dots, N,\ \forall i = 1, \dots, 4,
\end{aligned}$$

which gives the optimal state and input trajectory $\mathbf{x}^{\mathcal{P}_1*} = (x_1^{\mathcal{P}_1*}, \dots, x_N^{\mathcal{P}_1*})^T$ and $\mathbf{u}^{\mathcal{P}_1*} = (u_0^{\mathcal{P}_1*}, \dots, u_{N-1}^{\mathcal{P}_1*})^T$.

While $u_0^{\mathcal{P}_1*}$ is fed to platoon $\mathcal{P}_1$ as control input, $\mathbf{x}^{\mathcal{P}_1*}$ is used consensually in the constraint of the tracking MPC formulation for platoon $\mathcal{P}_2$, given as follows:

$$\begin{aligned}
\min_{x^{\mathcal{P}_2[1]}, u^{\mathcal{P}_2[1]}} \quad & \sum_{k=0}^{N-1} \|x_k^{\mathcal{P}_2[1]} - x_{ss}^{\mathcal{P}_2[1]}\|_{Q_1}^2 + \|\bar{d}_k - \bar{d}_{ss}\|_{Q_d}^2 \\
& + \|u_k^{\mathcal{P}_2[1]}\|_{R_1}^2 && \text{(17a)} \\
\text{s.t.} \quad & x_{k+1}^{\mathcal{P}_2[1]} = A_{11}^{\mathcal{P}_2} x_k^{\mathcal{P}_2[1]} + B_1^{\mathcal{P}_2} u_k^{\mathcal{P}_2[1]} && \text{(17b)} \\
& x_k^{\mathcal{P}_2[1]} \in \mathbb{X}^{\mathcal{P}_2[1]}, u_k^{\mathcal{P}_2[1]} \in \mathbb{U}^{\mathcal{P}_2[1]}, && \text{(17c)} \\
& \bar{d}_{k+1} = \bar{d}_k + v_k^{\mathcal{P}_1*[2]}\tau - v_k^{\mathcal{P}_2[1]}\tau, && \text{(17d)} \\
& x_0^{\mathcal{P}_2[1]} = x^{\mathcal{P}_2[1]}(t_t), \ x_N^{\mathcal{P}_2[1]} = x_{ss}^{\mathcal{P}_2[1]}, && \text{(17e)} \\
& \forall k = 0, \dots, N.
\end{aligned}$$

The P&P request is permitted only when both platoons $\mathcal{P}_1$ and $\mathcal{P}_2$ have reached their corresponding steady-states.

Remark IV.3. The terminal constraints in (16d) and (17e) can be further improved by enabling a neighborhood of the steady-state which allows for the P&P operation, i.e. $\|x_k - x_{ss}\|_2^2 \in \mathcal{B}, \ \mathcal{B} \in \mathbb{R}_+$. To preserve the recursive feasibility, (15) should now be solved using any state-of-the-art robust MPC technique, for example [22].

Theorem IV.1. Consider the linear dynamical systems of two platoons $\mathcal{P}_1$ and $\mathcal{P}_2$ and the P&P operation presented in Algorithm 1. If the P&P request is permitted, i.e., $\mathcal{P}_1$ and $\mathcal{P}_2$ are controlled to the steady-state given by (15). Then, starting from the steady-state, the closed loop system of the modified platoon $\tilde{\mathcal{P}}$ under the redesigned MPC control law and the redesigned dynamics of the terminal constraints obtained from (6) is asymptotically stable.

Proof. The proof follows directly from Theorem III.3 in [16]. The feasibility of the problem in (3) guarantees the stability of the modified closed loop system. The feasibility of the problem in (15) ensures that the modified MPC problem is feasible, when the new system starts from the steady-state. □

Remark IV.4. For simplicity of notations, in Algorithm 1 and Theorem IV.1 we only consider the sample platoon formation. However, both the proposed algorithm and the theoretical results can be directly applied to general cases.

V. Numerical Examples

In this section we illustrate the performance of our proposed controller with a simulation example where an HDV platoon handles the P&P request from a PV during its cruising on the highway.

In the simulation all HDVs in $\mathcal{P}_1$ are subject to the constraints, Velocity: $-2.0 \leqslant v^{\mathcal{P}1[i]} \leqslant 2.0$ m/s, Input torque: $-60300 \leqslant T^{\mathcal{P}1[i]} \leqslant 2200$ Nm. Likewise the constraints for the PV in $\mathcal{P}_2$ are, Orientation: $-0.2 \leqslant \varphi^{\mathcal{P}2[1]} \leqslant 0.2$ rad, Acceleration: $-3.0 \leqslant a^{\mathcal{P}2[1]} \leqslant 0.8$ m/s^2, Steering angle: $-0.08 \leqslant \delta^{\mathcal{P}2[1]} \leqslant 0.08$ rad. Note that those constraints are enforced on the states and inputs of the linearized systems.

A. Redesign Phase and the Formation Coordinator

First we demonstrate how the Formation Coordinator determines the plug-in location when the PV intends to join the HDV platoon.

Consider the platoon formations introduced in Remark III.1 where we have $\mathcal{P}_2 = \{\mathcal{Q}_1^{\text{PV}}\}$ sending plug-in request to $\mathcal{P}_1 = \{\mathcal{Q}_1^{\text{HDV}}, \dots, \mathcal{Q}_4^{\text{HDV}}\}$. The Formation Coordinator performs the controller redesign according to all possible plug-in options $\tilde{\mathcal{P}}_l = \mathbb{M}(\mathcal{P}_1, \mathcal{P}_2, l), \ l \in \{1, 2, 3\}$, giving the values $\alpha_1^* = 1.59 \times 10^{-3}$, $\alpha_2^* = 1.68 \times 10^{-3}$ and $\alpha_3^* = 0.33 \times 10^{-3}$. Consequently, the Formation Coordinator chooses 2 as the plug-in location since $\alpha_{max} = \alpha_2^*$.

B. Transition Phase and Plug-in Operation

With the plug-in location being chosen in Section V-A, the new platoon has the formation $\tilde{\mathcal{P}} = \mathbb{M}(\mathcal{P}_1, \mathcal{P}_2, 2) = \{\mathcal{Q}_1^{\text{HDV}}, \mathcal{Q}_2^{\text{HDV}}, \mathcal{Q}_1^{\text{PV}}, \mathcal{Q}_3^{\text{HDV}}, \mathcal{Q}_4^{\text{HDV}}\}$.

Before any platoon maneuver is enabled, we first compute the steady-states to be used in the transition phase. Based on formation $\tilde{\mathcal{P}}$, (15) is feasible, with the initial distance between $\mathcal{Q}_2^{\text{HDV}}$ and $\mathcal{Q}_1^{\text{PV}}$ given as $\bar{d}_{init} = 15$ m. Some of the resulting steady-states are listed as follows:

- Relative distance of $\mathcal{Q}_2^{\text{HDV}}$: $d_{ss}^{\mathcal{P}_1[2]} = -0.66$ m,
- Relative distance of $\mathcal{Q}_3^{\text{HDV}}$: $d_{ss}^{\mathcal{P}_1[3]} = 19.01$ m,
- Distance between $\mathcal{Q}_2^{\text{HDV}}$ and $\mathcal{Q}_1^{\text{PV}}$: $\bar{d}_{ss} = 15.13$ m.

We now show the simulation results of the transition phase and the subsequent plug-in operation. Note that the control law is applied to the linearized system.

Figure 3 shows a bird's eye view of the movements of all five vehicles. It can be observed that the relative distance between any two vehicles in a neighborhood are well-maintained in both phases. The transition phase takes 7.7 seconds to finish. Right after that, all vehicles are switched to the new control law $\tilde{\kappa}(\cdot)$, which enables the plug-in operation. $\mathcal{Q}_1^{\text{PV}}$ changes its lane and merges into $\mathcal{P}_1$. The extra room split by $\mathcal{Q}_2^{\text{HDV}}$ and $\mathcal{Q}_3^{\text{HDV}}$ ensures the feasibility of this maneuver. Finally all vehicles are regulated to their desired equilibria within 5 seconds. Note that for visibility of Figure 3, the vehicles are not plotted in their real scale.

Figure 4 shows the state and input trajectory of each vehicle subsystem, corresponding to their maneuvers presented in Figure 3. The significant jump in the state $d^{\mathcal{P}_1[3]}$ of $\mathcal{Q}_3^{\text{HDV}}$ at $t = 7.7$ is due to the state redefinition (14). After $\mathcal{Q}_1^{\text{PV}}$ has joined $\mathcal{P}_1$, all states in the new platoon $\tilde{\mathcal{P}}$ quickly converge to their equilibria, which shows that the asymptotic stability is achieved.

The simulation results demonstrate that the proposed controller successfully handles the P&P request and forms the vehicles into the desired platoon formation, with stability and feasibility being guaranteed at all times.

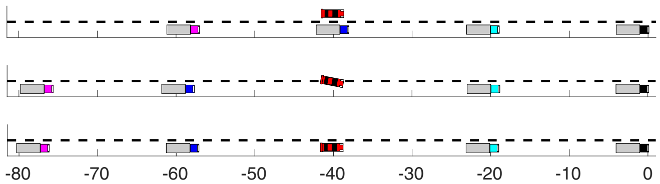

Fig. 3: Simulation of the transition phase and the plug-in operation. The upper subfigure indicates the position of the five vehicles at the beginning of the transition phase. At $t = 7.7s$, as all the vehicles arriving at their steady-states, they switch to the redesigned controller and the PV in red performs the plug-in operation, as shown by the middle subfigure. Finally at $t = 14.1s$, the P&P operation is completed as all the vehicles have been regulated to their equilibria in the new platoon.

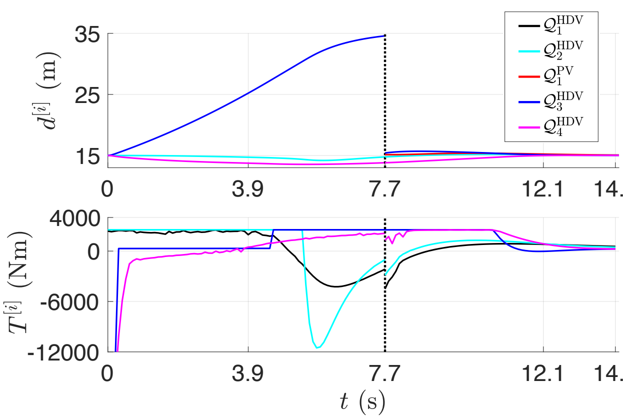

Fig. 4: Trajectories of $d^{\mathcal{P}_1[i]}$, $d^{\mathcal{P}_2[1]}$ and HDV inputs $T^{\mathcal{P}_1[i]}$. Black dashed lines mark the time $t = 7.7s$ when the transition phase is finished and the plug-in operation is carried out.

VI. Conclusions

In this paper we proposed a distributed MPC framework for HDV platooning, with the ability to handle P&P operations requested by passenger vehicles. First, we introduced the approach of Formation Coordinator which decides the platoon formation. Second, we developed a tailored transition phase that ensures a collision-free P&P operation. We proved that our method guarantees stability and feasibility at all times. The performance of the proposed controller has been demonstrated on a multi-vehicle platooning system.

Our future work will focus on exploring the robustness of the proposed controller, e.g. the string stability, when the controlled platoon is subject to external disturbances and communication delay.

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